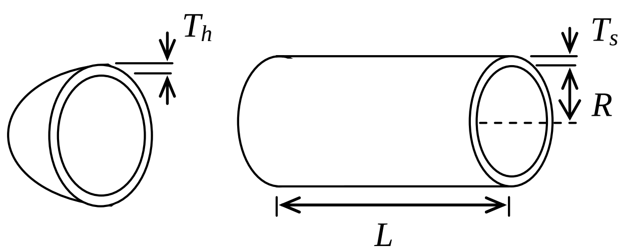
Pressure vessels

(Return to: Product Family Design Optimization Testbed)

Overview:

A pressure vessel is a cylindrical object with hemispherical heads at both ends, as shown below (Williams *et al.,* 2004). In the present example problem, the manufacturer forms platforms of the pressure vessels to maximize the average profit across the entire market space while providing necessary product variety.



Schematic of Pressure Vessel

The pressure vessel example was first incorporated by Hernandez (2001) to design product platforms. Williams (2003) mentioned in his work that "the pressure vessel is not only one of the familiar examples for every mechanical engineer but also needs few design equations for it's description. This helps in emphasizing on the problem and solution rather than the methodology used to solve it. Thus, the methodology will not be case-specific."

In the present case example, it is assumed that the product is in the detail design stage, where the pressure, volume, thickness of shell plate, thickness of head plate and the radii of shell and head are to be calculated. It is assumed that there is no performance loss of the products being produced since the manufacturer caters to the technical specifications as required by the market. Thus, the technical specifications act as constraints and the product platform is designed by maximizing the total profit of all variants in the product family.

Mathematical Description:

Nomenclature:

The nomenclature used in the present problem is as follows:

A – Scaling coefficient of the Gaussian function

C_s – Cost per kg of processed shell steel (assumed \$0.8/kg)

C_h – Cost per kg of forged steel for the head (assumed \$2/kg)

$C_{s,raw}$ – Cost per Kg of raw steel plate (assumed \$0.3/kg)

C_w – Cost per Kg of welding material (assumed \$15/kg)

C_o – Raw material ordering cost (assumed \$250)

C_V – Cost constant for volume (assumed \$185/ m^3)

C_P – Cost Constant for pressure (assumed \$185/ m^3)

CM – Cost of material utilized in each pressure vessel
CW – Cost of welding for each pressure vessel
$C(L, R, T_S, T_h)$ - Sum of material and welding cost for each pressure vessel
D_i – Demand of specific product variant *i*
D_{tot} – Total demand of products in the market space
L – Length of shell (in m)
$L_{s,raw}$ – Length of raw material (in m)
m - Total number of distinct raw material sheets required
n – Total number of product variants
N_P – Number of presses required
O – Objective function
P – Pressure (in MPa)
R – Radius of shell and head (in m)
T_s - Thickness of shell plate (m)
T_h - Thickness of head plate (in m)
V – Volume (in m^3)
q – Number of discrete steps (Demand values)
μ - Mean of demand distribution
σ - Standard deviation of demand distribution
σ_y - Yield Strength (assumed 1077 MPa)
ρ_d - Density of material (assumed 7800 kg/m^3)

Design variables:
In the present problem, pressure (P) and volume (V) are performance variables of the pressure vessel, which are treated as independent variables. The other design variables namely, the thicknesses and radii of shell plate and head plate, and the length of the pressure vessel are calculated using Equations 1 to 3 listed below. The limits of the market space for the pressure vessel are given in terms of the performance measures: Pressure (P) (ranging from 10 MPa to 30 MPa) and Volume (V) (ranging from 10m^3 to 30m^3).

$$T_S \geq \left(\frac{P}{\sigma y - 0.6P}\right)R \quad \ldots (1)$$

$$T_h \geq \left(\frac{P}{2\sigma y - 0.2P}\right)R \quad \ldots (2)$$

The
length of pressure vessel is calculated by the following equation:

$$V = \pi R^2 L + \frac{4}{3}\pi R^3 \quad \ldots (3)$$

Demand scenarios:

Ten different non-uniform demand scenarios as considered by Williams *et al.* (2004) are shown in Table 1. The demand for each of these ten scenarios is taken as input to form a product platform.

The non-uniform demand scenarios shown in Table 1 are modeled by the following four methods, as discussed by Williams (2003).

a) Discrete function

For discrete demand, the objective function is given by

$$O = \sum_{i}^{q} \frac{1}{D_i} \sum_{i=y_{i.\min}}^{y_{i.\max}} \sum_{j=x_{i.\min}}^{x_{i.\max}} .... \sum_{k=n_{i.\min}}^{n_{i.\max}} o(x_j, y_i, ...n_k) \qquad \text{...(4)}$$

where *O* denotes the objective function value, *q* denotes the total number of discrete steps(demand values) in the market space and *max* and *min* subscripts representing the upper and lower bounds of each discrete step.

b) Linear continuous function

For the case of linear demand, the objective function is given by

$$O = \sum_{i=y_{\min}}^{y_{\max}} \sum_{j=x_{\min}}^{x_{\max}} .... \sum_{k=n_{\min}}^{n_{\max}} \frac{1}{D(x_j, y_i, ...n_k)} o(x_j, y_i, ...n_k) \qquad \text{...(5)}$$

where *D(x,y)* is in the form,

$$D(x, y, ...n) = ax + by + ... + cn + d \qquad \text{...(6)}$$

where a, b, c and d are coefficients that are used to make approximation of the linear trend of the demand.

c) n-Dimensional Gaussian distribution

One-dimensional Gaussian distribution is given by the equation below:

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad \text{...(7)}$$

where *μ* is the mean of the distribution at which maximum demand occurs, and σ is the standard deviation of the distribution. The demand for an *n*-dimensional market space is represented by the function given by equation

$$D(x, y) = \prod_{i}^{n} \left(A_i \frac{1}{\sigma_i \sqrt{2\pi}} e^{\frac{1}{2}\left(\frac{i-\mu_i}{\sigma_i}\right)^2} \right) \qquad \text{...(8)}$$

where subscript *i* represents the statistical properties corresponding to each individual dimension and variable *A* represents a scaling coefficient used to translate the probability of receiving orders for a product to the demand of that product. The objective function for the present case is similar to Equation (5), with the demand function replaced by Equation (8).

Table 1: Ten Demand Scenarios (adopted from Williams *et al.,* 2004)

Demand Scenario	Description	Max. Demand	Min. Demand	Demand model method
Discrete Pyramid Large (DPL)	Depicts large discrete changes in demand, majority of demand in the centre of market space	400 vessels	150 vessels	Discrete function
Discrete Pyramid Small (DPS)	Depicts discrete, discontinuous changes in demand	300 vessels	0 vessels	Discrete function
Linear Large (LL)	Depicts linearity in demand. Demand increases from lower-end to higher-end vessels	600 vessels	200 vessels	Linear continuous function
Linear Small (LS)	Similar to LL but with lower value of demand	60 vessels	20 vessels	Linear continuous function
Normal Centre Small (NCS)	Depicts low demand normal distributed with the mean in the centre of the market space	450 vessels	0 vessels	Normal distribution
Normal High (NH)	Depicts demand normal distributed with the mean on the lower-end vessels	700 vessels	0 vessels	Normal distribution
Normal Low (NL)	Depicts demand normal distributed with the mean on the Higher-end vessels	700 vessels	0 vessels	Normal distribution
Normal Centre Large (NCL)	Depicts high demand normal distributed with the mean in the centre of the market space	1200 vessels	0 vessels	Normal distribution
Random Large (RL)	Depicts high demand random in nature	500 vessels	0 vessels	Random distribution
Random Small (RS)	Depicts low demand random in nature	10 vessels	0 vessels	Random distribution

d) Random distribution

For the case of random distribution, the objective function is similar to Equation (5). However, the demand of each individual product variant is unique.

For each of the ten demand scenarios, the values of the following design variables are calculated using Equations 1 to 3 by considering the related technical constraints.

a) Thickness of shell plate *(T_s)*,
b) Thickness of head plate *(T_h)*,
c) Radius of shell and head *(R).*

Cost Modeling (Hernandez, 2001):
The total manufacturing cost of a pressure vessel is taken as the sum of material cost, welding cost, order cost, and equipment cost (ignoring all labor and plant costs).

a) The material cost is given as the sum of the cost of material actually used in each vessel and the cost of the raw material wasted by cutting the raw steel plates to the required dimensions. The cost of material utilized in each pressure vessel is computed as follows:

$$CM = 2\pi\rho_d (C_s R T_s L + C_h R^2 T_h + C_{s,raw} R(L_{s,raw} - L)) \qquad \ldots(9)$$

b) The total cost of welding is given by the following equation,

$$CW = 2\pi\rho_d \left(\frac{2}{9} C_w T_s^2 L + \frac{4}{9} C_w \pi T_s^2 R\right) \qquad \ldots(10)$$

Thus, the sum of material and welding costs is represented by the function,

$$C(L, R, T_S, T_h) = CM + CW \qquad \ldots(11)$$

c) A cost of $250 is levied on each order of raw material to account for shipping, handling and stocking in inventory. This cost is dependent on number of different sheet sizes of raw material ordered and is independent of the quantity of sheets. This cost is denoted as,

$$C_o = \$250 \qquad \ldots(12)$$

d) The equipment cost of the forging press and associated dies for manufacturing the heads is approximated as,

$$EquipCost = \sum_{P=1}^{Np} (500000 + 50000 R_P) \qquad \ldots(13)$$

Equation (11) represents the cost of producing one pressure vessel. Equation (12) and (13) are calculated for the entire family of pressure vessels. It is assumed that one forging press of each radius is capable of meeting any demand.

The selling price of each pressure vessel is assumed to be a function of pressure and volume and is given by,

$$Sellingpri\ ce = C_V V + C_P P \qquad \ldots(14)$$

The total profit is given by the summation of differences of the selling price and cost of manufacturing of all product variants, *n*, as well as subtracting the ordering cost of raw materials and the equipment cost.

$$profit_{total} = \sum_{i}^{n} \left[Sellingprice_i - C_i(L, R, T_S, T_h) \right] - C_o - EquipCost \qquad \ldots(15)$$

Problem Formulation:

Common and scaling values of design variables of the pressure vessel product family are identified. A scale-based platform that allows the design of required product variants with highest average profit is formed. This platform is further optimized by identifying possible modular combinations for maximum average profit of the product variants in the entire market place.

The final objective is to maximize the average total profit of pressure vessels in the entire market space. The average total profit is the summation of profit from each product variant of the entire market space divided by the total demand of the market space. The objective function is given as follows (Williams, 2003):

$$Avgprofit = \frac{1}{D_{tot}} \left[\left(\sum_{i=1}^{n} D_i \left[Sellingprice_i - C_i(L, R, T_S, T_h) \right] \right) - C_o - EquipCost \right] \ldots(16)$$

The same objective function has been incorporated by other researchers who have used the case example of pressure vessels, like Hernandez *et al* (2001, 2003), Carone *et al.* (2003), Williams *et al.* (2004), Kulkarni *et al.* (2005), and Borrappa and Allada (2006).

The technical constraints are as follows:

1. Limits on the maximum radius as 1.5 m, and the maximum length as 7m, constrained by the equipment available with the manufacturer.
2. Design constraints of the pressure vessel on the minimum allowable thickness of its shell and head, as given in Equations 2 and 3.
3. Design constraints governing modular combinations.
4. Carbon steel ASME SA 203 Grade B as a raw material for producing pressure vessels, available in thicknesses ranging between 6.35 mm (0.25 in) and 76.2 mm (3 in).)

Classification:

The problems that used pressure vessels as the case example have been classified as A2/B2, based on the classification scheme presented in the 'Product Family Design Optimization Testbed'. The justification for the classification is as follows:

A2 is the category in which it is not specified which design variables are to be used as platform variables. The common variables and the ranges of scaling variables across different products have been determined by the methodology of the problem. B2 is the category in which explicit demand model (here, for non-uniform demand) is used to derive the demand information of different variants from the platform. However, competition has not been taken into account.

References:

Based on the two criteria for classification mentioned in the Product Family Design Optimization Testbed classification scheme, we assign code scheme to each of the product platform problems that use the family of pressure vessels as the case study. This classification is presented in Table 2 below. The table also includes analysis based on the type of demand incorporated, trade-off issues and the consideration of uncertainty factor.

Table 2: Differentiation of different references that use pressure vessel as a case example

No.	Reference	Demand	Uncertainty	Trade-off	Code scheme
1.	Hernandez (2001), Hernandez *et al.* (2003)	Uniform	Not considered	Cost and variety	A2/B2
2.	Carone *et al.* (2003)	Uniform	Considered (design)	Cost and variety	A2/B2
3.	Williams *et al.* (2004)	Non-uniform	Not considered	Platform extent and performance	A2/B2
4.	Kulkarni *et al.* (2005)	Non-uniform	Considered (Demand)	Profit and deviation in profit due to fluctuation in demand	A2/B2
5.	Borappa and Allada (2006)	Non-uniform	Not considered	Platform extent and performance	A2/B2

Comments and Discussion:

The present case example of Pressure vessels was introduced by Hernandez (2001) to illustrate the design of multiple hybrid product platforms. In the later years, other researches have extended the problem scope by considering non-uniform demand, uncertainty in market space, etc. as shown in Table 2 above. The cost model, technical constraints and objective function formulation have been maintained the same across all the papers. All the works consider the demand model without competition and determination of platform variables, and hence are categorized as A2/B2. One of the papers (Borappa and Allada, 2006) benchmarks the results obtained by Williams *et al.* (2004).

The scope of the present problem can be extended to B3, by considering competition. Uncertainty in other parameters like customer needs, technology, competition, etc. and consideration of planning horizon can further enrich the problem scope.

MATLAB© Code:

Program Files

```
%-----------------------------------------------------------------------
```

Demand Data Generation Module

```
%-----------------------------------------------------------------------
clc
clear all
%PHASE 1: Demand Data Generation Module
%Choose a demand scenario
fprintf('        Demand Data Generation Module \n
****************************************\n\n\n\n');
fprintf('Please change the "output/input filename" destination in the
"DemandGenerationModule.m" MATLAB to suit your preferences\n\n\n\n');
x=input('Choose a demand scenario from the following:\n(Relevant demand
data is provided in an excel sheet by name Demandinfo on the
desktop)\n1.   Discrete Pyramid Large\n2.  Discrete Pyramid Small\n3.
Linear Large\n4.    Linear Small \n5.   Normal Centre Small\n6. Normal
High \n7.    Normal Low \n8. Normal Centre Large\n9. Random Large\n10.
Random Small \n:-');
a=1;
% Discrete Pyramid Large
if (x==1)
    i=0;
for v=10:30
    for p=10:30
    D=150;
    if (v >=14 && v <= 26 ) && (p >=14 && p <= 26 )
             D=200;
           matrix(v,p)=D;
     end

    if (v >=15 && v <= 25) && (p >=15 && p <= 25)
          D=250;
           matrix(v,p)=D;
    end

    if (v >=17 && v <= 23) && (p >=17 && p <= 23)
           D=300;
           matrix(v,p)=D;
   end

    if (v >=18 && v <= 22) && (p >=18 && p <= 22)
             D=350;
           matrix(v,p)=D;
   end

   if (v >=19 && v <= 21) && (p >=19 && p <= 21)
           D=400;
           matrix(v,p)=D;
   end
```

```
   Demandpat(i+1,1)=v;
   Demandpat(i+1,2)=p;
   Demandpat(i+1,3)=D;
   i=i+1;

    end
end

map =[1 1 1];
colormap(gray);
lighting flat;
mesh(matrix)
xlabel('Volume (m^3)')
ylabel('Pressure (MPa)')
zlabel('Demand (# of vessels)')
headings1 = {'Volume' 'Pressure' 'Demand'};
xlswrite('s:\desktop\Demandinfo.xls',headings1,'dpL','A1')
xlswrite('s:\desktop\Demandinfo.xls',Demandpat,'dpL','A2')
% Discrete Pyramid Small
elseif (x==2)
        i=0;
for v=10:30
    for p=10:30
    D=0;
    if (v >=14 && v <= 26 ) && (p >=14 && p <= 26 )
             D=50;
           matrix(v,p)=D;
     end

    if (v >=15 && v <= 25) && (p >=15 && p <= 25)
          D=100;
           matrix(v,p)=D;
    end

    if (v >=17 && v <= 23) && (p >=17 && p <= 23)
           D=150;
           matrix(v,p)=D;
   end

    if (v >=18 && v <= 22) && (p >=18 && p <= 22)
             D=250;
           matrix(v,p)=D;
   end

   if (v >=19 && v <= 21) && (p >=19 && p <= 21)
           D=300;
           matrix(v,p)=D;
   end


   Demandpat(i+1,1)=v;
   Demandpat(i+1,2)=p;
   Demandpat(i+1,3)=D;
   i=i+1;
```

```
    end
end

map = [1 1 1];
colormap(map);
figure(2);
mesh(matrix)
xlabel('Volume (m^3)')
ylabel('Pressure (MPa)')
zlabel('Demand (# of vessels)')
headings1 = {'Volume' 'Pressure' 'Demand'};
xlswrite('s:\desktop\Demandinfo.xls',headings1,'dps','A1')
xlswrite('s:\desktop\Demandinfo.xls',Demandpat,'dps','A2')
% Linear Large
    elseif (x==3)
i=10;
j=10;
for p=1:1:441
        Demandpat(p,1) = i;
        Demandpat(p,2) = j;
        Demandpat(p,3)=round (15*i+7.5*j);
        j=j+1;
        if j==31
            i=i+1;
            j=10;
        end
end
headings1 = {'Volume' 'Pressure' 'Demand'};
xlswrite('s:\desktop\Demandinfo.xls',headings1,'LL','A1')
xlswrite('s:\desktop\Demandinfo.xls',Demandpat,'LL','A2')
% Linear Small
        elseif (x==4)
i=10;
j=10;
for p=1:1:441
        Demandpat(p,1) = i;
        Demandpat(p,2) = j;
        Demandpat(p,3)=round (15*i/10+7.5*j/10);
        j=j+1;
        if j==31
            i=i+1;
            j=10;
        end
end
headings1 = {'Volume' 'Pressure' 'Demand'};
xlswrite('s:\desktop\Demandinfo.xls',headings1,'LS','A1')
xlswrite('s:\desktop\Demandinfo.xls',Demandpat,'LS','A2')
%  Normal Centre Small
elseif (x==5)
i=10;
j=10;
xm=20;
xv=15;
xsd=sqrt(xv);
```

```
ym=20;
yv=15;
ysd=sqrt(yv);
for p=1:1:441

        Demandpat(p,1) = i;
        Demandpat(p,2) = j;
        xD=200*(1/(xsd*sqrt(2*pi)))*(exp(-0.5*((i-xm)/xsd)*((i-
xm)/xsd)));
        yD=200*(1/(ysd*sqrt(2*pi)))*(exp(-0.5*((j-ym)/ysd)*((j-
ym)/ysd)));
        Demandpat(p,3)=round(xD*yD);

        j=j+1;
        if j==31
            i=i+1;
            j=10;
        end
end
headings1 = {'Volume' 'Pressure' 'Demand'};
xlswrite('s:\desktop\Demandinfo.xls',headings1,'NCS','A1')
xlswrite('s:\desktop\Demandinfo.xls',Demandpat,'NCS','A2')
plot3(Demandpat(:,1),Demandpat(:,2),Demandpat(:,3))
% Normal High
elseif (x==6)
                i=10;
j=10;
xm=12;
xv=15;
xsd=sqrt(xv);
ym=12;
yv=5;
ysd=sqrt(yv);
for p=1:1:441

        Demandpat(p,1) = i;
        Demandpat(p,2) = j;
        xD=200*(1/(xsd*sqrt(2*pi)))*(exp(-0.5*((i-xm)/xsd)*((i-
xm)/xsd)));
        yD=200*(1/(ysd*sqrt(2*pi)))*(exp(-0.5*((j-ym)/ysd)*((j-
ym)/ysd)));
        Demandpat(p,3)=round(xD*yD);

        j=j+1;
        if j==31
            i=i+1;
            j=10;
        end
end
headings1 = {'Volume' 'Pressure' 'Demand'};
xlswrite('s:\desktop\Demandinfo.xls',headings1,'NH','A1')
xlswrite('s:\desktop\Demandinfo.xls',Demandpat,'NH','A2')
plot3(Demandpat(:,1),Demandpat(:,2),Demandpat(:,3))
%  Normal Low
elseif (x==7)
                        i=10;
```

```
j=10;
xm=24;
xv=15;
xsd=sqrt(xv);
ym=24;
yv=5;
ysd=sqrt(yv);
for p=1:1:441

        Demandpat(p,1) = i;
        Demandpat(p,2) = j;
        xD=200*(1/(xsd*sqrt(2*pi)))*(exp(-0.5*((i-xm)/xsd)*((i-
xm)/xsd)));
        yD=200*(1/(ysd*sqrt(2*pi)))*(exp(-0.5*((j-ym)/ysd)*((j-
ym)/ysd)));
        Demandpat(p,3)=round(xD*yD);

        j=j+1;
        if j==31
            i=i+1;
            j=10;
        end
end
headings1 = {'Volume' 'Pressure' 'Demand'};
xlswrite('s:\desktop\Demandinfo.xls',headings1,'NL','A1')
xlswrite('s:\desktop\Demandinfo.xls',Demandpat,'NL','A2')
plot3(Demandpat(:,1),Demandpat(:,2),Demandpat(:,3))
% Normal Centre Large
elseif (x==8)
                                i=10;
j=10;
xm=20;
xv=5;
xsd=sqrt(xv);
ym=20;
yv=5;
ysd=sqrt(yv);
for p=1:1:441

        Demandpat(p,1) = i;
        Demandpat(p,2) = j;
        xD=200*(1/(xsd*sqrt(2*pi)))*(exp(-0.5*((i-xm)/xsd)*((i-
xm)/xsd)));
        yD=200*(1/(ysd*sqrt(2*pi)))*(exp(-0.5*((j-ym)/ysd)*((j-
ym)/ysd)));
        Demandpat(p,3)=round(xD*yD);

        j=j+1;
        if j==31
            i=i+1;
            j=10;
        end
end
headings1 = {'Volume' 'Pressure' 'Demand'};
xlswrite('s:\desktop\Demandinfo.xls',headings1,'NCL','A1')
xlswrite('s:\desktop\Demandinfo.xls',Demandpat,'NCL','A2')
```

```
plot3(Demandpat(:,1),Demandpat(:,2),Demandpat(:,3))

% Random Large
elseif (x==9)
i=10;
j=10;
for p=1:1:441
        Demandpat(p,1) = i;
        Demandpat(p,2) = j;
        Demandpat(p,3)=round(500*random('beta',1,[1]));
        j=j+1;
        if j==31
            i=i+1;
            j=10;
        end
end
headings1 = {'Volume' 'Pressure' 'Demand'};
xlswrite('s:\desktop\Demandinfo.xls',headings1,'RL','A1')
xlswrite('s:\desktop\Demandinfo.xls',Demandpat,'RL','A2')

% Random Small
elseif (x==10)
i=10;
j=10;
for p=1:1:441
        Demandpat(p,1) = i;
        Demandpat(p,2) = j;
        Demandpat(p,3)=round(10*random('beta',1,[1]));
        j=j+1;
        if j==31
            i=i+1;
            j=10;
        end
end
headings1 = {'Volume' 'Pressure' 'Demand'};
xlswrite('s:\desktop\Demandinfo.xls',headings1,'RS','A1')
xlswrite('s:\desktop\Demandinfo.xls',Demandpat,'RS','A2')
end %End of Demand pattern
x=input('\n\n(Relevant demand data is provided in an excel sheet by
name "Demandinfo.xls" on the desktop)\n');
%----------------------------------------------------------------------
```

Technical Feasibility Checker

```
%----------------------------------------------------------------------
%PHASE 2: Techincal Constarints Feasibility Checker Module
clc
clear all
a=1;
%initializing the constants
row_den=7800;%Density of Material in Kg/m^3
sig_y=1077;%Yield strength in MPa
Cs=0.8; %Cost/Kg of processed shell steel
Ch=2;%Cost/Kg of the forged steel for the head
Cw=15;%Cost/Kg for the welding material
Csraw=.3;%Cost/Kg of raw steel plate
Cv=185;%Cost constant for volume
Cp=185;%Cost constant for pressure
```

```
% In this program, we read the designsolutionsdata.xls file, where we
have
% a sample of pressure vessel variants having different values of
Pressure, Volume, Shell Thickness, Head
% Thickness, Length of the Shell, and Radius. The program is designed
to remove technically infeasible product variants
% and generate an output file called "Feasible.xls" that consists of
technically feasible solutions only.
% We use the technical constraint equations provided by thesis of
Williams (2003) to code the technical constraint equations.

fprintf('  Techincal Constarints Feasibility Checker Module
\n**************************************\n\n\n');
fprintf('Please change the "output/input filename" destination in the
"TechincalConstarintsFeasibilityCheckerModule.m" MATLAB to suit your
preferences\n\n\n\n');
fprintf('\n We will check if the product variant parameters are
technically feasible in this module.\n The product variant parameters\n
(whose values have to be checked for technical feasibility while
exploring the design space)\n are as follows:
1.Pressure(pressure)\n2.Volume(volume)\n3.Radius(r)\n4.Raw material
length(Lraw)\n5.Shell thickness(Ts)\n6.Head thickness(Th)')
Feasibilitycheck=xlsread('s:\desktop\designsolutionsdata.xls','Feasibil
itydata')
[m,n]=size (Feasibilitycheck)
for i=1:m
   for j=1:n
        if (j==1)
            if (Feasibilitycheck (i,j)>=10 &&
Feasibilitycheck(i,j)<=30)
            else
                break;
            end
        end
        if (j==2)
            if (Feasibilitycheck (i,j)>=10 &&
Feasibilitycheck(i,j)<=30)
            else
                break;
            end
        end
          if (j==3)
%           Ltemp(i)= ((Feasibilitycheck (i,1)/(3.14*Feasibilitycheck
(i,j)^2))- ((4/3)*Feasibilitycheck (i,j)));
                if (Feasibilitycheck (i,j)>0 && Feasibilitycheck
(i,j)<=1.5)
%                       && Ltemp(i)>0 && Ltemp(i)<=7)
%                 elseif (Feasibilitycheck (i,j)<0 || Feasibilitycheck
(i,j)>=1.5)
%                       break;
%                 elseif (Ltemp(i)<0 || Ltemp(i)>7)
                else
                    break;
                end
    end
```

```
  if (j==4)

            if (Feasibilitycheck (i,j)>0 && Feasibilitycheck(i,j)<=7)

            else
                break;
            end
  end
  if (j==5)
            if (Feasibilitycheck (i,j)>=((Feasibilitycheck
(i,2)/(sig_y-0.6*Feasibilitycheck (i,2)))*Feasibilitycheck (i,5)))
            else
                break;
            end
        end
         if (j==6)
            if (Feasibilitycheck (i,j)>=((Feasibilitycheck
(i,2)/(sig_y-0.6*Feasibilitycheck (i,2)))*Feasibilitycheck (i,5)))
               frow(a)=i;
                a=a+1;
            else
                break;
            end
        end

    end
end

[p,q]=size(frow);
for b=1:q
    for g=1:6
    FeasibleMatrix(b,g)=Feasibilitycheck(frow(b),g);
    end
end
headings = {'Volume' 'Pressure' 'Radius' 'Length' 'Shell thickness'
'Head thickness' };
xlswrite('s:\desktop\Feasible.xls',headings,'Data','A1')
xlswrite('s:\desktop\Feasible.xls',FeasibleMatrix,'Data','A2')

%-----------------------------------------------------------------------
```

Cost Calculator

```
% ----------------------------------------------------------------------
%PHASE 3: Average Profit of the Product Family Calculator Module

%initializing the constants
clc
clear all

row_den=7800;%Density of Material in Kg/m^3
sig_y=1077;%Yield strength in MPa
Cs=0.8; %Cost/Kg of processed shell steel
Ch=2;%Cost/Kg of the forged steel for the head
Cw=15;%Cost/Kg for the welding material
Csraw=.3;%Cost/Kg of raw steel plate
Cv=185;%Cost constant for volume
```

```
Cp=185;%Cost constant for pressure



fprintf('     Average Profit of the Product Family Calculator
Module\n****************************************************\n\n')
fprintf('Please change the "output/input filename" destination in the
"AverageProfitCalculator.m" MATLAB to suit your preferences\n\n\n\n');
fprintf('Please enter the results file from the optimization technique,
naming it as finalresults.xls\n(Sample template is enclosed for the
results of Borappa and Allada, 2006):-');
a = xlsread('s:\desktop\Input Phase 3.xls','NH');
[w,z]=size(a);
%calculating average profit of a Pressue Vessel from the formed
platform

Cost_total=0;
for i=1:w
 Cost(i) = 2*pi*row_den*(Cs*a(i,3)*a(i,5)*a(i,4) +
Ch*a(i,3)*a(i,3)*a(i,6) + 2/9*Cw*a(i,5)*a(i,5)*a(i,4) +
4/9*Cw*pi*a(i,5)*a(i,5)*a(i,3) + Csraw*a(i,5)*a(i,3)*(a(i,8)-a(i,4)));
% Material Cost and Welding Cost calculation
 SP(i)=Cv*a(i,1)+Cp*a(i,2); %Selling Price of each product
 Cost_SP(i) = a(i,9)*(SP(i)-Cost(i)); %Total Profit of each product =
Profit of each product*Demand of each product
Cost_total=Cost_SP(i) + Cost_total; %Total profit over the product
family.
end

rad_uniq = unique(a(:,3));
[r1,r2]=size(rad_uniq);
rad_cost=0;
for p=1:(r1-1)
    if abs(rad_uniq(p)-rad_uniq(p+1))<= 0.01
     rad_cost= [rad_cost;rad_uniq(p+1)];
    else
        rad_cost=[rad_cost;rad_uniq(p)];
    end
end
rad_cost=[rad_cost;rad_uniq(r1)];

[i1,i2]=size(rad_cost);
rad_uniq = rad_cost((2:(i1)),:);

[r1,r2]=size(rad_uniq);%rad_uniq is the set of unique radius for the
family of products
equip_cost=0;
for l=1:r1
    equip_cost = [equip_cost;500000+50000*rad_uniq(l)];%Equipment Cost
end

[d,e]=size(equip_cost);
total_equip_cost=0;
for m=1:d
```

```
    total_equip_cost=total_equip_cost+equip_cost(m);%Total Equipment
Cost
end

sheets = unique(a(:,8));
[s1,s2]=size(sheets); % s1 is the number of distinct sheets of metal
required
Co = s1*250; % Each time an order for raw materail is placed a fee of
$250 is applied for shipping charges

demand = a(:,9);
[d1,d2]=size(demand);
total_demand=0;
for g=1:d1
    total_demand=total_demand + demand(g);%Total Demand of the Family
of Products
end
total_demand;
Cost_total;
Co ;
total_equip_cost;
cost_avg = (1/total_demand)*(Cost_total - Co -
total_equip_cost);%Calculation of the Average Profit for the family of
products

for k=1:w
    a(k,10) = cost_avg(1);
end


headings = {'Volume' 'Pressure' 'Radius' 'Length' 'Shell thickness'
'Head thickness'  'Vmax' 'Lraw' 'Demand' 'Avg_Profit'};
xlswrite('s:\desktop\Output Phase 3.xls',headings,'results','A1')
xlswrite('s:\desktop\Output Phase 3.xls' , a, 'results', 'A2')

% ----------------------------------------------------------------------
```

Input Files

As mentioned in the MATLAB© codes above, two Excel files are given as input: one is “designsolutiondata.xls” given as input to the the Technical Feasibility Checker module and the other is “Input Phase 3.xls” given as input to the Cost Calculator module. The input data is provided hereunder:

Data from “designsolutionsdata.xls”

Volume	Pressure	Radius	Length	Shell thickness	Head thickness
10	10	0.8005	3.902567	0.00748	0.00372
10	10	0.8005	9	0.00748	0.0092
10	40	0.8005	3.902567	0.00748	0.00372
90	10	0.8005	3.902567	0.00748	0.00372
10	11	0.8018	3.88473082	0	0.00412
10	60	0.8023	3.87789158	0.009	0.00448
10	13	1.5	3.89707059	0.00976	0.00484
10	14	0.8029	3.86969971	0.01052	0.00524
10	15	6	3.90669419	0.01124	0.0056
20	16	0.8005	3.902567	0.012	0.00596
10	17	0.8007	3.89981787	0.01276	0.00636
17	18	8	3.898444	0.01352	0.00672
10	19	0.8008	3.898444	0.01428	0.00708
10	20	0.8008	3.898444	0.01504	0.00748
40	21	0.8008	3.898444	0.0158	0.00784
10	22	0.8007	3.89981787	0.01656	0.0082
10	23	0.8006	3.9011922	0.01732	0.0086
11	24	0.8004	3.90394226	0.01808	0.00896
10	25	0.8003	3.905318	0.01884	0.00932
10	26	0.8001	3.90807085	0.0196	0.00972
18	24	0.8412	6.97950627	0.019	0.0094
18	25	0.8411	6.98156603	0.0198	0.0098
18	26	0.8409	6.98568761	0.0206	0.0102
18	27	0.8403	6.99806887	0.0214	0.01056
18	28	0.8405	6.99393903	0.0222	0.01096
18	29	0.8403	6.99806887	0.023	0.01136
18	30	0.8415	6.9733311	0.02384	0.01176
22	10	0.9253	6.94955603	0.00864	0.00432
22	11	0.9233	6.98771345	0.00952	0.00472
22	12	0.9235	6.98388733	0.01036	0.00516
22	13	0.9244	6.96669837	0.01124	0.0056
26	27	0.9979	6.98460837	0.0254	0.01256
26	28	0.9974	6.99361394	0.02636	0.013
26	29	0.9982	6.97921104	0.02732	0.01348
26	30	0.9982	6.97921104	0.02828	0.01396
30	10	1.0667	6.974402	0.00996	0.00496
30	11	1.0665	6.97781821	0.01096	0.00548
30	12	1.0662	6.98294584	0.01196	0.00596
30	13	1.0657	6.99150077	0.01296	0.00644
30	14	1.0655	6.99492585	0.01396	0.00696
30	15	1.0668	6.97269457	0.015	0.00744
30	16	1.0674	6.96245923	0.016	0.00796

Data from "Input Phase 3.xls"

Volume	Pressure	Radius	Length	Shell thickness	Head thickness	Volumemax	Lraw	Demand
10	10	0.8005	3.902567	0.00748	0.00372	10	3.908070854	431
10	11	0.8018	3.884730816	0.00824	0.00412	10	3.908070854	582
10	12	0.8023	3.877891578	0.009	0.00448	10	3.908070854	643
10	13	0.8009	3.89707059	0.00976	0.00484	10	3.908070854	582
10	14	0.8029	3.869699709	0.01052	0.00524	10	3.908070854	431
10	15	0.8002	3.906694192	0.01124	0.0056	10	3.908070854	262
10	16	0.8005	3.902567	0.012	0.00596	10	3.908070854	130
10	17	0.8007	3.899817865	0.01276	0.00636	10	3.908070854	53
10	18	0.8008	3.898443996	0.01352	0.00672	10	3.908070854	18
10	19	0.8008	3.898443996	0.01428	0.00708	10	3.908070854	5
10	20	0.8008	3.898443996	0.01504	0.00748	10	3.908070854	1
10	21	0.8008	3.898443996	0.0158	0.00784	10	3.908070854	0
10	22	0.8007	3.899817865	0.01656	0.0082	10	3.908070854	0
10	23	0.8006	3.9011922	0.01732	0.0086	10	3.908070854	0
10	24	0.8004	3.903942265	0.01808	0.00896	10	3.908070854	0
10	25	0.8003	3.905317995	0.01884	0.00932	10	3.908070854	0
10	26	0.8001	3.908070854	0.0196	0.00972	10	3.908070854	0
10	27	0.8014	3.890210527	0.0204	0.01008	10	3.908070854	0
10	28	0.801	3.89569765	0.02116	0.01044	10	3.908070854	0
10	29	0.8009	3.89707059	0.02192	0.01084	10	3.908070854	0
10	30	0.8006	3.9011922	0.02268	0.0112	10	3.908070854	0
11	10	0.8005	5.890527133	0.00748	0.00372	14	6.989811941	477
11	11	0.8018	5.866249809	0.00824	0.00412	14	6.989811941	643
11	12	0.8023	5.856941542	0.009	0.00448	14	6.989811941	711
11	13	0.8009	5.883045493	0.00976	0.00484	14	6.989811941	643
11	14	0.8029	5.845792926	0.01052	0.00524	14	6.989811941	477
11	15	0.8002	5.896145202	0.01124	0.0056	14	6.989811941	289
11	16	0.8005	5.890527133	0.012	0.00596	14	6.989811941	144
11	17	0.8007	5.886785011	0.01276	0.00636	14	6.989811941	58
11	18	0.8008	5.884914927	0.01352	0.00672	14	6.989811941	19
11	19	0.8008	5.884914927	0.01428	0.00708	14	6.989811941	5
11	20	0.8008	5.884914927	0.01504	0.00748	14	6.989811941	1
11	21	0.8008	5.884914927	0.0158	0.00784	14	6.989811941	0
11	22	0.8007	5.886785011	0.01656	0.0082	14	6.989811941	0
11	23	0.8006	5.888655746	0.01732	0.0086	14	6.989811941	0
11	24	0.8004	5.89239917	0.01808	0.00896	14	6.989811941	0
11	25	0.8003	5.89427186	0.01884	0.00932	14	6.989811941	0
11	26	0.8001	5.898019196	0.0196	0.00972	14	6.989811941	0
11	27	0.8014	5.873708071	0.0204	0.01008	14	6.989811941	0
11	28	0.801	5.88117671	0.02116	0.01044	14	6.989811941	0
11	29	0.8009	5.883045493	0.02192	0.01084	14	6.989811941	0
11	30	0.8006	5.888655746	0.02268	0.0112	14	6.989811941	0

Volume	Pressure	Radius	Length	Shell thickness	Head thickness	Volumemax	Lraw	Demand
12	14	0.842	6.963052885	0.01104	0.00548	14	6.999205705	493
12	15	0.843	6.942547728	0.01184	0.00588	14	6.999205705	299
12	16	0.8405	6.993939029	0.0126	0.00628	14	6.999205705	148
12	17	0.8408	6.98774943	0.0134	0.00668	14	6.999205705	60
12	18	0.841	6.983626473	0.0142	0.00704	14	6.999205705	20
12	19	0.8412	6.979506265	0.015	0.00744	14	6.999205705	5
12	20	0.8413	6.977447192	0.0158	0.00784	14	6.999205705	1
12	21	0.8413	6.977447192	0.0166	0.00824	14	6.999205705	0
12	22	0.8413	6.977447192	0.0174	0.00864	14	6.999205705	0
12	23	0.841	6.983626473	0.0182	0.009	14	6.999205705	0
12	24	0.8412	6.979506265	0.019	0.0094	14	6.999205705	0
12	25	0.8411	6.981566025	0.0198	0.0098	14	6.999205705	0
12	26	0.8409	6.985687607	0.0206	0.0102	14	6.999205705	0
12	27	0.8403	6.998068874	0.0214	0.01056	14	6.999205705	0
12	28	0.8405	6.993939029	0.0222	0.01096	14	6.999205705	0
12	29	0.8403	6.998068874	0.023	0.01136	14	6.999205705	0
12	30	0.8415	6.973331105	0.02384	0.01176	14	6.999205705	0
13	10	0.9253	6.949556034	0.00864	0.00432	14	6.999205705	477
13	11	0.9233	6.987713451	0.00952	0.00472	14	6.999205705	643
13	12	0.9235	6.98388733	0.01036	0.00516	14	6.999205705	711
13	13	0.9244	6.966698365	0.01124	0.0056	14	6.999205705	643
13	14	0.9251	6.95336142	0.01212	0.00604	14	6.999205705	477
13	15	0.9227	6.999205705	0.01296	0.00644	14	6.999205705	289
13	16	0.9233	6.987713451	0.01384	0.00688	14	6.999205705	144
13	17	0.9237	6.980063521	0.01472	0.00732	14	6.999205705	58
13	18	0.924	6.974332137	0.0156	0.00776	14	6.999205705	19
13	19	0.9234	6.985800101	0.01648	0.00816	14	6.999205705	5
13	20	0.9244	6.966698365	0.01736	0.0086	14	6.999205705	1
13	21	0.9245	6.964791362	0.01824	0.00904	14	6.999205705	0
13	22	0.9245	6.964791362	0.01912	0.00948	14	6.999205705	0
13	23	0.9233	6.987713451	0.02	0.00988	14	6.999205705	0
13	24	0.9241	6.97242283	0.02088	0.01032	14	6.999205705	0
13	25	0.9243	6.968605944	0.02176	0.01076	14	6.999205705	0
13	26	0.9242	6.970514099	0.02264	0.0112	14	6.999205705	0
13	27	0.9231	6.991541886	0.02352	0.0116	14	6.999205705	0
13	28	0.9238	6.978152482	0.0244	0.01204	14	6.999205705	0
13	29	0.9236	6.981975136	0.02528	0.01248	14	6.999205705	0
13	30	0.9234	6.985800101	0.02616	0.01292	14	6.999205705	0
14	10	0.9981	6.981009648	0.00932	0.00464	14	6.999205705	431
14	11	1.0003	6.941555526	0.01028	0.00512	14	6.999205705	582
14	12	0.9984	6.975615321	0.0112	0.0056	14	6.999205705	643
14	13	0.9995	6.95587458	0.01216	0.00604	14	6.999205705	582
14	14	0.9983	6.97741293	0.01308	0.00652	14	6.999205705	431
14	15	0.998	6.982808758	0.01404	0.00696	14	6.999205705	262
14	16	0.998	6.982808758	0.01496	0.00744	14	6.999205705	130
14	17	0.999	6.964840161	0.01592	0.00792	14	6.999205705	53

Volume	Pressure	Radius	Length	Shell thickness	Head thickness	Volumemax	Lraw	Demand
14	18	0.9974	6.9936139	0.01684	0.00836	14	6.99902331	18
14	19	0.9982	6.979211	0.0178	0.00884	14	6.99902331	5
14	20	0.9976	6.9900102	0.01876	0.00928	14	6.99902331	1
14	21	0.9974	6.9936139	0.01968	0.00976	14	6.99902331	0
14	22	0.998	6.9828088	0.02064	0.01024	14	6.99902331	0
14	23	0.998	6.9828088	0.0216	0.01068	14	6.99902331	0
14	24	0.9988	6.9684299	0.02256	0.01116	14	6.99902331	0
14	25	0.9971	6.9990233	0.02348	0.0116	14	6.99902331	0
14	26	0.9977	6.9882091	0.02444	0.01208	14	6.99902331	0
14	27	0.9979	6.9846084	0.0254	0.01256	14	6.99902331	0
14	28	0.9974	6.9936139	0.02636	0.013	14	6.99902331	0
14	29	0.9982	6.979211	0.02732	0.01348	14	6.99902331	0
14	30	0.9982	6.979211	0.02828	0.01396	18	6.99902331	0
15	10	1.0667	6.974402	0.00996	0.00496	18	6.998352705	365
15	11	1.0665	6.9778182	0.01096	0.00548	18	6.998352705	493
15	12	1.0662	6.9829458	0.01196	0.00596	18	6.998352705	545
15	13	1.0657	6.9915008	0.01296	0.00644	18	6.998352705	493
15	14	1.0655	6.9949258	0.01396	0.00696	18	6.998352705	365
15	15	1.0668	6.9726946	0.015	0.00744	18	6.998352705	221
15	16	1.0674	6.9624592	0.016	0.00796	18	6.998352705	110
15	17	1.0667	6.974402	0.017	0.00844	18	6.998352705	45
15	18	1.0656	6.9932131	0.018	0.00892	18	6.998352705	15
15	19	1.0655	6.9949258	0.019	0.00944	18	6.998352705	4
15	20	1.0664	6.979527	0.02004	0.00992	18	6.998352705	1
15	21	1.0664	6.979527	0.02104	0.01044	18	6.998352705	0
15	22	1.0657	6.9915008	0.02204	0.01092	18	6.998352705	0
15	23	1.0653	6.9983527	0.02308	0.0114	18	6.998352705	0
15	24	1.0661	6.9846559	0.02408	0.01192	18	6.998352705	0
15	25	1.0653	6.9983527	0.02508	0.0124	18	6.998352705	0
15	26	1.0662	6.9829458	0.02612	0.01292	18	6.998352705	0
15	27	1.0655	6.9949258	0.02712	0.0134	18	6.998352705	0
15	28	1.0662	6.9829458	0.02816	0.01392	18	6.998352705	0
15	29	1.0654	6.9966391	0.02916	0.0144	18	6.998352705	0
15	30	1.0654	6.9966391	0.0302	0.01488	18	6.998352705	0
16	10	0.8005	3.902567	0.00748	0.00372	18	6.998352705	289
16	11	0.8018	3.8847308	0.00824	0.00412	18	6.998352705	390
16	12	0.8023	3.8778916	0.009	0.00448	18	6.998352705	431
16	13	0.8009	3.8970706	0.00976	0.00484	18	6.998352705	390
16	14	0.8029	3.8696997	0.01052	0.00524	18	6.998352705	289
16	15	0.8002	3.9066942	0.01124	0.0056	18	6.998352705	175
16	16	0.8005	3.902567	0.012	0.00596	18	6.998352705	87
16	17	0.8007	3.8998179	0.01276	0.00636	18	6.998352705	35
16	18	0.8008	3.898444	0.01352	0.00672	18	6.998352705	12
16	19	0.8008	3.898444	0.01428	0.00708	18	6.998352705	3
16	20	0.8008	3.898444	0.01504	0.00748	18	6.998352705	1
16	21	0.8008	3.898444	0.0158	0.00784	18	6.998352705	0

Volume	Pressure	Radius	Length	Shell thickness	Head thickness	Volumemax	Lraw	Demand
16	22	0.8007	3.8998179	0.01656	0.0082	18	6.993939029	0
16	23	0.8006	3.9011922	0.01732	0.0086	18	6.993939029	0
16	24	0.8004	3.9039423	0.01808	0.00896	18	6.993939029	0
16	25	0.8003	3.905318	0.01884	0.00932	18	6.993939029	0
16	26	0.8001	3.9080709	0.0196	0.00972	18	6.993939029	0
16	27	0.8014	3.8902105	0.0204	0.01008	18	6.993939029	0
16	28	0.801	3.8956976	0.02116	0.01044	18	6.993939029	0
16	29	0.8009	3.8970706	0.02192	0.01084	18	6.993939029	0
16	30	0.8006	3.9011922	0.02268	0.0112	18	6.993939029	0
17	10	0.8005	5.8905271	0.00748	0.00372	18	6.993939029	214
17	11	0.8018	5.8662498	0.00824	0.00412	18	6.993939029	289
17	12	0.8023	5.8569415	0.009	0.00448	18	6.993939029	319
17	13	0.8009	5.8830455	0.00976	0.00484	18	6.993939029	289
17	14	0.8029	5.8457929	0.01052	0.00524	18	6.993939029	214
17	15	0.8002	5.8961452	0.01124	0.0056	18	6.993939029	130
17	16	0.8005	5.8905271	0.012	0.00596	18	6.993939029	65
17	17	0.8007	5.886785	0.01276	0.00636	18	6.993939029	26
17	18	0.8008	5.8849149	0.01352	0.00672	18	6.993939029	9
17	19	0.8008	5.8849149	0.01428	0.00708	18	6.993939029	2
17	20	0.8008	5.8849149	0.01504	0.00748	18	6.993939029	1
17	21	0.8008	5.8849149	0.0158	0.00784	18	6.993939029	0
17	22	0.8007	5.886785	0.01656	0.0082	18	6.993939029	0
17	23	0.8006	5.8886557	0.01732	0.0086	18	6.993939029	0
17	24	0.8004	5.8923992	0.01808	0.00896	18	6.993939029	0
17	25	0.8003	5.8942719	0.01884	0.00932	18	6.993939029	0
17	26	0.8001	5.8980192	0.0196	0.00972	18	6.993939029	0
17	27	0.8014	5.8737081	0.0204	0.01008	18	6.993939029	0
17	28	0.801	5.8811767	0.02116	0.01044	18	6.993939029	0
17	29	0.8009	5.8830455	0.02192	0.01084	18	6.993939029	0
17	30	0.8006	5.8886557	0.02268	0.0112	18	6.993939029	0
18	10	0.8435	6.9323207	0.00788	0.00392	18	6.993939029	148
18	11	0.8407	6.9898119	0.00864	0.00432	18	6.993939029	200
18	12	0.8415	6.9733311	0.00944	0.00472	18	6.993939029	221
18	13	0.8407	6.9898119	0.01024	0.00508	18	6.993939029	200
18	14	0.842	6.9630529	0.01104	0.00548	18	6.993939029	148
18	15	0.843	6.9425477	0.01184	0.00588	18	6.993939029	90
18	16	0.8405	6.993939	0.0126	0.00628	18	6.993939029	45
18	17	0.8408	6.9877494	0.0134	0.00668	18	6.993939029	18
18	18	0.841	6.9836265	0.0142	0.00704	18	6.993939029	6
18	19	0.8412	6.9795063	0.015	0.00744	18	6.993939029	2
18	20	0.8413	6.9774472	0.0158	0.00784	18	6.993939029	0
18	21	0.8413	6.9774472	0.0166	0.00824	18	6.993939029	0
18	22	0.8413	6.9774472	0.0174	0.00864	18	6.993939029	0
18	23	0.841	6.9836265	0.0182	0.009	18	6.993939029	0
18	24	0.8412	6.9795063	0.019	0.0094	18	6.993939029	0
18	25	0.8411	6.981566	0.0198	0.0098	18	6.993939029	0

Volume	Pressure	Radius	Length	Shell thickness	Head thickness	Volumemax	Lraw	Demand
18	26	0.8409	6.985687607	0.0206	0.0102	18	6.999205705	0
18	27	0.8403	6.998068874	0.0214	0.01056	18	6.999205705	0
18	28	0.8405	6.993939029	0.0222	0.01096	18	6.999205705	0
18	29	0.8403	6.998068874	0.023	0.01136	18	6.999205705	0
18	30	0.8415	6.973331105	0.02384	0.01176	18	6.999205705	0
19	10	0.9253	6.949556034	0.00864	0.00432	22	6.999205705	96
19	11	0.9233	6.987713451	0.00952	0.00472	22	6.999205705	130
19	12	0.9235	6.98388733	0.01036	0.00516	22	6.999205705	144
19	13	0.9244	6.966698365	0.01124	0.0056	22	6.999205705	130
19	14	0.9251	6.95336142	0.01212	0.00604	22	6.999205705	96
19	15	0.9227	6.999205705	0.01296	0.00644	22	6.999205705	58
19	16	0.9233	6.987713451	0.01384	0.00688	22	6.999205705	29
19	17	0.9237	6.980063521	0.01472	0.00732	22	6.999205705	12
19	18	0.924	6.974332137	0.0156	0.00776	22	6.999205705	4
19	19	0.9234	6.985800101	0.01648	0.00816	22	6.999205705	1
19	20	0.9244	6.966698365	0.01736	0.0086	22	6.999205705	0
19	21	0.9245	6.964791362	0.01824	0.00904	22	6.999205705	0
19	22	0.9245	6.964791362	0.01912	0.00948	22	6.999205705	0
19	23	0.9233	6.987713451	0.02	0.00988	22	6.999205705	0
19	24	0.9241	6.97242283	0.02088	0.01032	22	6.999205705	0
19	25	0.9243	6.968605944	0.02176	0.01076	22	6.999205705	0
19	26	0.9242	6.970514099	0.02264	0.0112	22	6.999205705	0
19	27	0.9231	6.991541886	0.02352	0.0116	22	6.999205705	0
19	28	0.9238	6.978152482	0.0244	0.01204	22	6.999205705	0
19	29	0.9236	6.981975136	0.02528	0.01248	22	6.999205705	0
19	30	0.9234	6.985800101	0.02616	0.01292	22	6.999205705	0
20	10	0.9981	6.981009648	0.00932	0.00464	22	6.999205705	58
20	11	1.0003	6.941555526	0.01028	0.00512	22	6.999205705	79
20	12	0.9984	6.975615321	0.0112	0.0056	22	6.999205705	87
20	13	0.9995	6.95587458	0.01216	0.00604	22	6.999205705	79
20	14	0.9983	6.97741293	0.01308	0.00652	22	6.999205705	58
20	15	0.998	6.982808758	0.01404	0.00696	22	6.999205705	35
20	16	0.998	6.982808758	0.01496	0.00744	22	6.999205705	18
20	17	0.999	6.964840161	0.01592	0.00792	22	6.999205705	7
20	18	0.9974	6.993613943	0.01684	0.00836	22	6.999205705	2
20	19	0.9982	6.979211039	0.0178	0.00884	22	6.999205705	1
20	20	0.9976	6.990010209	0.01876	0.00928	22	6.999205705	0
20	21	0.9974	6.993613943	0.01968	0.00976	22	6.999205705	0
20	22	0.998	6.982808758	0.02064	0.01024	22	6.999205705	0
20	23	0.998	6.982808758	0.0216	0.01068	22	6.999205705	0
20	24	0.9988	6.968429883	0.02256	0.01116	22	6.999205705	0
20	25	0.9971	6.99902331	0.02348	0.0116	22	6.999205705	0
20	26	0.9977	6.988209094	0.02444	0.01208	22	6.999205705	0
20	27	0.9979	6.984608369	0.0254	0.01256	22	6.999205705	0
20	28	0.9974	6.993613943	0.02636	0.013	22	6.999205705	0
20	29	0.9982	6.979211039	0.02732	0.01348	22	6.999205705	0

Volume	Pressure	Radius	Length	Shell thickness	Head thickness	Volumemax	Lraw	Demand
20	30	0.9982	6.979211039	0.02828	0.01396	22	6.998352705	0
21	10	1.0667	6.974402005	0.00996	0.00496	22	6.998352705	33
21	11	1.0665	6.977818209	0.01096	0.00548	22	6.998352705	45
21	12	1.0662	6.98294584	0.01196	0.00596	22	6.998352705	49
21	13	1.0657	6.991500766	0.01296	0.00644	22	6.998352705	45
21	14	1.0655	6.994925846	0.01396	0.00696	22	6.998352705	33
21	15	1.0668	6.972694567	0.015	0.00744	22	6.998352705	20
21	16	1.0674	6.962459226	0.016	0.00796	22	6.998352705	10
21	17	1.0667	6.974402005	0.017	0.00844	22	6.998352705	4
21	18	1.0656	6.993213083	0.018	0.00892	22	6.998352705	1
21	19	1.0655	6.994925846	0.019	0.00944	22	6.998352705	0
21	20	1.0664	6.979526976	0.02004	0.00992	22	6.998352705	0
21	21	1.0664	6.979526976	0.02104	0.01044	22	6.998352705	0
21	22	1.0657	6.991500766	0.02204	0.01092	22	6.998352705	0
21	23	1.0653	6.998352705	0.02308	0.0114	22	6.998352705	0
21	24	1.0661	6.984655938	0.02408	0.01192	22	6.998352705	0
21	25	1.0653	6.998352705	0.02508	0.0124	22	6.998352705	0
21	26	1.0662	6.98294584	0.02612	0.01292	22	6.998352705	0
21	27	1.0655	6.994925846	0.02712	0.0134	22	6.998352705	0
21	28	1.0662	6.98294584	0.02816	0.01392	22	6.998352705	0
21	29	1.0654	6.996639053	0.02916	0.0144	22	6.998352705	0
21	30	1.0654	6.996639053	0.0302	0.01488	22	6.998352705	0
22	10	0.8005	3.902567	0.00748	0.00372	22	6.998352705	18
22	11	0.8018	3.884730816	0.00824	0.00412	22	6.998352705	24
22	12	0.8023	3.877891578	0.009	0.00448	22	6.998352705	26
22	13	0.8009	3.89707059	0.00976	0.00484	22	6.998352705	24
22	14	0.8029	3.869699709	0.01052	0.00524	22	6.998352705	18
22	15	0.8002	3.906694192	0.01124	0.0056	22	6.998352705	11
22	16	0.8005	3.902567	0.012	0.00596	22	6.998352705	5
22	17	0.8007	3.899817865	0.01276	0.00636	22	6.998352705	2
22	18	0.8008	3.898443996	0.01352	0.00672	22	6.998352705	1
22	19	0.8008	3.898443996	0.01428	0.00708	22	6.998352705	0
22	20	0.8008	3.898443996	0.01504	0.00748	22	6.998352705	0
22	21	0.8008	3.898443996	0.0158	0.00784	22	6.998352705	0
22	22	0.8007	3.899817865	0.01656	0.0082	22	6.998352705	0
22	23	0.8006	3.9011922	0.01732	0.0086	22	6.998352705	0
22	24	0.8004	3.903942265	0.01808	0.00896	22	6.998352705	0
22	25	0.8003	3.905317995	0.01884	0.00932	22	6.998352705	0
22	26	0.8001	3.908070854	0.0196	0.00972	22	6.998352705	0
22	27	0.8014	3.890210527	0.0204	0.01008	22	6.998352705	0
22	28	0.801	3.89569765	0.02116	0.01044	22	6.998352705	0
22	29	0.8009	3.89707059	0.02192	0.01084	22	6.998352705	0
22	30	0.8006	3.9011922	0.02268	0.0112	22	6.998352705	0
23	10	0.8005	5.890527133	0.00748	0.00372	26	5.890527133	9
23	11	0.8018	5.866249809	0.00824	0.00412	26	5.890527133	12
23	12	0.8023	5.856941542	0.009	0.00448	26	5.890527133	13

Volume	Pressure	Radius	Length	Shell thickness	Head thickness	Volumemax	Lraw	Demand
23	13	0.8009	5.883045493	0.00976	0.00484	26	6.999205705	12
23	14	0.8029	5.845792926	0.01052	0.00524	26	6.999205705	9
23	15	0.8002	5.896145202	0.01124	0.0056	26	6.999205705	5
23	16	0.8005	5.890527133	0.012	0.00596	26	6.999205705	3
23	17	0.8007	5.886785011	0.01276	0.00636	26	6.999205705	1
23	18	0.8008	5.884914927	0.01352	0.00672	26	6.999205705	0
23	19	0.8008	5.884914927	0.01428	0.00708	26	6.999205705	0
23	20	0.8008	5.884914927	0.01504	0.00748	26	6.999205705	0
23	21	0.8008	5.884914927	0.0158	0.00784	26	6.999205705	0
23	22	0.8007	5.886785011	0.01656	0.0082	26	6.999205705	0
23	23	0.8006	5.888655746	0.01732	0.0086	26	6.999205705	0
23	24	0.8004	5.89239917	0.01808	0.00896	26	6.999205705	0
23	25	0.8003	5.89427186	0.01884	0.00932	26	6.999205705	0
23	26	0.8001	5.898019196	0.0196	0.00972	26	6.999205705	0
23	27	0.8014	5.873708071	0.0204	0.01008	26	6.999205705	0
23	28	0.801	5.88117671	0.02116	0.01044	26	6.999205705	0
23	29	0.8009	5.883045493	0.02192	0.01084	26	6.999205705	0
23	30	0.8006	5.888655746	0.02268	0.0112	26	6.999205705	0
24	10	0.8435	6.932320709	0.00788	0.00392	26	6.999205705	4
24	11	0.8407	6.989811941	0.00864	0.00432	26	6.999205705	5
24	12	0.8415	6.973331105	0.00944	0.00472	26	6.999205705	6
24	13	0.8407	6.989811941	0.01024	0.00508	26	6.999205705	5
24	14	0.842	6.963052885	0.01104	0.00548	26	6.999205705	4
24	15	0.843	6.942547728	0.01184	0.00588	26	6.999205705	2
24	16	0.8405	6.993939029	0.0126	0.00628	26	6.999205705	1
24	17	0.8408	6.98774943	0.0134	0.00668	26	6.999205705	0
24	18	0.841	6.983626473	0.0142	0.00704	26	6.999205705	0
24	19	0.8412	6.979506265	0.015	0.00744	26	6.999205705	0
24	20	0.8413	6.977447192	0.0158	0.00784	26	6.999205705	0
24	21	0.8413	6.977447192	0.0166	0.00824	26	6.999205705	0
24	22	0.8413	6.977447192	0.0174	0.00864	26	6.999205705	0
24	23	0.841	6.983626473	0.0182	0.009	26	6.999205705	0
24	24	0.8412	6.979506265	0.019	0.0094	26	6.999205705	0
24	25	0.8411	6.981566025	0.0198	0.0098	26	6.999205705	0
24	26	0.8409	6.985687607	0.0206	0.0102	26	6.999205705	0
24	27	0.8403	6.998068874	0.0214	0.01056	26	6.999205705	0
24	28	0.8405	6.993939029	0.0222	0.01096	26	6.999205705	0
24	29	0.8403	6.998068874	0.023	0.01136	26	6.999205705	0
24	30	0.8415	6.973331105	0.02384	0.01176	26	6.999205705	0
25	10	0.9253	6.949556034	0.00864	0.00432	26	6.999205705	2
25	11	0.9233	6.987713451	0.00952	0.00472	26	6.999205705	2
25	12	0.9235	6.98388733	0.01036	0.00516	26	6.999205705	3
25	13	0.9244	6.966698365	0.01124	0.0056	26	6.999205705	2
25	14	0.9251	6.95336142	0.01212	0.00604	26	6.999205705	2
25	15	0.9227	6.999205705	0.01296	0.00644	26	6.999205705	1
25	16	0.9233	6.987713451	0.01384	0.00688	26	6.999205705	1

Volume	Pressure	Radius	Length	Shell thickness	Head thickness	Volumemax	Lraw	Demand
25	17	0.9237	6.980063521	0.01472	0.00732	26	6.99902331	0
25	18	0.924	6.974332137	0.0156	0.00776	26	6.99902331	0
25	19	0.9234	6.985800101	0.01648	0.00816	26	6.99902331	0
25	20	0.9244	6.966698365	0.01736	0.0086	26	6.99902331	0
25	21	0.9245	6.964791362	0.01824	0.00904	26	6.99902331	0
25	22	0.9245	6.964791362	0.01912	0.00948	26	6.99902331	0
25	23	0.9233	6.987713451	0.02	0.00988	26	6.99902331	0
25	24	0.9241	6.97242283	0.02088	0.01032	26	6.99902331	0
25	25	0.9243	6.968605944	0.02176	0.01076	26	6.99902331	0
25	26	0.9242	6.970514099	0.02264	0.0112	26	6.99902331	0
25	27	0.9231	6.991541886	0.02352	0.0116	26	6.99902331	0
25	28	0.9238	6.978152482	0.0244	0.01204	26	6.99902331	0
25	29	0.9236	6.981975136	0.02528	0.01248	26	6.99902331	0
25	30	0.9234	6.985800101	0.02616	0.01292	26	6.99902331	0
26	10	0.9981	6.981009648	0.00932	0.00464	26	6.99902331	1
26	11	1.0003	6.941555526	0.01028	0.00512	26	6.99902331	1
26	12	0.9984	6.975615321	0.0112	0.0056	26	6.99902331	1
26	13	0.9995	6.95587458	0.01216	0.00604	26	6.99902331	1
26	14	0.9983	6.97741293	0.01308	0.00652	26	6.99902331	1
26	15	0.998	6.982808758	0.01404	0.00696	26	6.99902331	0
26	16	0.998	6.982808758	0.01496	0.00744	26	6.99902331	0
26	17	0.999	6.964840161	0.01592	0.00792	26	6.99902331	0
26	18	0.9974	6.993613943	0.01684	0.00836	26	6.99902331	0
26	19	0.9982	6.979211039	0.0178	0.00884	26	6.99902331	0
26	20	0.9976	6.990010209	0.01876	0.00928	26	6.99902331	0
26	21	0.9974	6.993613943	0.01968	0.00976	26	6.99902331	0
26	22	0.998	6.982808758	0.02064	0.01024	26	6.99902331	0
26	23	0.998	6.982808758	0.0216	0.01068	26	6.99902331	0
26	24	0.9988	6.968429883	0.02256	0.01116	26	6.99902331	0
26	25	0.9971	6.99902331	0.02348	0.0116	26	6.99902331	0
26	26	0.9977	6.988209094	0.02444	0.01208	26	6.99902331	0
26	27	0.9979	6.984608369	0.0254	0.01256	26	6.99902331	0
26	28	0.9974	6.993613943	0.02636	0.013	26	6.99902331	0
26	29	0.9982	6.979211039	0.02732	0.01348	26	6.99902331	0
26	30	0.9982	6.979211039	0.02828	0.01396	26	6.99902331	0
27	10	1.0667	6.974402005	0.00996	0.00496	30	6.994925846	0
27	11	1.0665	6.977818209	0.01096	0.00548	30	6.994925846	0
27	12	1.0662	6.98294584	0.01196	0.00596	30	6.994925846	0
27	13	1.0657	6.991500766	0.01296	0.00644	30	6.994925846	0
27	14	1.0655	6.994925846	0.01396	0.00696	30	6.994925846	0
27	15	1.0668	6.972694567	0.015	0.00744	30	6.994925846	0
27	16	1.0674	6.962459226	0.016	0.00796	30	6.994925846	0
27	17	1.0667	6.974402005	0.017	0.00844	30	6.994925846	0
27	18	1.0656	6.993213083	0.018	0.00892	30	6.994925846	0
27	19	1.0655	6.994925846	0.019	0.00944	30	6.994925846	0
27	20	1.0664	6.979526976	0.02004	0.00992	30	6.994925846	0

Volume	Pressure	Radius	Length	Shell thickness	Head thickness	Volumemax	Lraw	Demand
27	21	1.0664	6.979526976	0.02104	0.01044	30	6.998352705	0
27	22	1.0657	6.991500766	0.02204	0.01092	30	6.998352705	0
27	23	1.0653	6.998352705	0.02308	0.0114	30	6.998352705	0
27	24	1.0661	6.984655938	0.02408	0.01192	30	6.998352705	0
27	25	1.0653	6.998352705	0.02508	0.0124	30	6.998352705	0
27	26	1.0662	6.98294584	0.02612	0.01292	30	6.998352705	0
27	27	1.0655	6.994925846	0.02712	0.0134	30	6.998352705	0
27	28	1.0662	6.98294584	0.02816	0.01392	30	6.998352705	0
27	29	1.0654	6.996639053	0.02916	0.0144	30	6.998352705	0
27	30	1.0654	6.996639053	0.0302	0.01488	30	6.998352705	0
28	10	0.8005	3.902567	0.00748	0.00372	30	6.998352705	0
28	11	0.8018	3.884730816	0.00824	0.00412	30	6.998352705	0
28	12	0.8023	3.877891578	0.009	0.00448	30	6.998352705	0
28	13	0.8009	3.89707059	0.00976	0.00484	30	6.998352705	0
28	14	0.8029	3.869699709	0.01052	0.00524	30	6.998352705	0
28	15	0.8002	3.906694192	0.01124	0.0056	30	6.998352705	0
28	16	0.8005	3.902567	0.012	0.00596	30	6.998352705	0
28	17	0.8007	3.899817865	0.01276	0.00636	30	6.998352705	0
28	18	0.8008	3.898443996	0.01352	0.00672	30	6.998352705	0
28	19	0.8008	3.898443996	0.01428	0.00708	30	6.998352705	0
28	20	0.8008	3.898443996	0.01504	0.00748	30	6.998352705	0
28	21	0.8008	3.898443996	0.0158	0.00784	30	6.998352705	0
28	22	0.8007	3.899817865	0.01656	0.0082	30	6.998352705	0
28	23	0.8006	3.9011922	0.01732	0.0086	30	6.998352705	0
28	24	0.8004	3.903942265	0.01808	0.00896	30	6.998352705	0
28	25	0.8003	3.905317995	0.01884	0.00932	30	6.998352705	0
28	26	0.8001	3.908070854	0.0196	0.00972	30	6.998352705	0
28	27	0.8014	3.890210527	0.0204	0.01008	30	6.998352705	0
28	28	0.801	3.89569765	0.02116	0.01044	30	6.998352705	0
28	29	0.8009	3.89707059	0.02192	0.01084	30	6.998352705	0
28	30	0.8006	3.9011922	0.02268	0.0112	30	6.998352705	0
29	10	0.8005	5.890527133	0.00748	0.00372	30	6.998352705	0
29	11	0.8018	5.866249809	0.00824	0.00412	30	6.998352705	0
29	12	0.8023	5.856941542	0.009	0.00448	30	6.998352705	0
29	13	0.8009	5.883045493	0.00976	0.00484	30	6.998352705	0
29	14	0.8029	5.845792926	0.01052	0.00524	30	6.998352705	0
29	15	0.8002	5.896145202	0.01124	0.0056	30	6.998352705	0
29	16	0.8005	5.890527133	0.012	0.00596	30	6.998352705	0
29	17	0.8007	5.886785011	0.01276	0.00636	30	6.998352705	0
29	18	0.8008	5.884914927	0.01352	0.00672	30	6.998352705	0
29	19	0.8008	5.884914927	0.01428	0.00708	30	6.998352705	0
29	20	0.8008	5.884914927	0.01504	0.00748	30	6.998352705	0
29	21	0.8008	5.884914927	0.0158	0.00784	30	6.998352705	0
29	22	0.8007	5.886785011	0.01656	0.0082	30	6.998352705	0
29	23	0.8006	5.888655746	0.01732	0.0086	30	6.998352705	0
29	24	0.8004	5.89239917	0.01808	0.00896	30	6.998352705	0

Volume	Pressure	Radius	Length	Shell thickness	Head thickness	Volumemax	Lraw	Demand
29	25	0.8003	5.89427186	0.01884	0.00932	30	6.998068874	0
29	26	0.8001	5.898019196	0.0196	0.00972	30	6.998068874	0
29	27	0.8014	5.873708071	0.0204	0.01008	30	6.998068874	0
29	28	0.801	5.88117671	0.02116	0.01044	30	6.998068874	0
29	29	0.8009	5.883045493	0.02192	0.01084	30	6.998068874	0
29	30	0.8006	5.888655746	0.02268	0.0112	30	6.998068874	0
30	10	0.8435	6.932320709	0.00788	0.00392	30	6.998068874	0
30	11	0.8407	6.989811941	0.00864	0.00432	30	6.998068874	0
30	12	0.8415	6.973331105	0.00944	0.00472	30	6.998068874	0
30	13	0.8407	6.989811941	0.01024	0.00508	30	6.998068874	0
30	14	0.842	6.963052885	0.01104	0.00548	30	6.998068874	0
30	15	0.843	6.942547728	0.01184	0.00588	30	6.998068874	0
30	16	0.8405	6.993939029	0.0126	0.00628	30	6.998068874	0
30	17	0.8408	6.98774943	0.0134	0.00668	30	6.998068874	0
30	18	0.841	6.983626473	0.0142	0.00704	30	6.998068874	0
30	19	0.8412	6.979506265	0.015	0.00744	30	6.998068874	0
30	20	0.8413	6.977447192	0.0158	0.00784	30	6.998068874	0
30	21	0.8413	6.977447192	0.0166	0.00824	30	6.998068874	0
30	22	0.8413	6.977447192	0.0174	0.00864	30	6.998068874	0
30	23	0.841	6.983626473	0.0182	0.009	30	6.998068874	0
30	24	0.8412	6.979506265	0.019	0.0094	30	6.998068874	0
30	25	0.8411	6.981566025	0.0198	0.0098	30	6.998068874	0
30	26	0.8409	6.985687607	0.0206	0.0102	30	6.998068874	0
30	27	0.8403	6.998068874	0.0214	0.01056	30	6.998068874	0
30	28	0.8405	6.993939029	0.0222	0.01096	30	6.998068874	0
30	29	0.8403	6.998068874	0.023	0.01136	30	6.998068874	0
30	30	0.8415	6.973331105	0.02384	0.01176	30	6.998068874	0

NOTE:

Determination of L_{raw} has been done differently by the researchers in the literature.

1. C. B. Williams (MS Thesis, 2003) – L_{raw} was computed based on the V_{max} (Maximum Volume) obtained by the optimization method and the corresponding r_{min} (Minimum Radius) by the expression:

$$L_{raw} = \frac{V_{\max}}{\pi \times r_{\min}^{2}} - \frac{4}{3} \times r_{\min}$$

2. Borappa and Allada (2006) - L_{raw} has been taken for the 1st L encountered for V_{max} determined by the optimization method. This heuristic assumes the corresponding r as the r_{min} and recalculates all the L for different products. Hence, L_{raw} is the maximum among all the L's for a given Vmax. It is important to mention that choice of the heuristic to select L_{raw} is dependent on the user-specific.